

is of course dependent on the estimate of η (the propulsive efficiency), which is usually 0.85 from V_m to $V_* = (H/G)^{1/4}$ (the velocity for the airplane's maximum lift-drag ratio). It is shown in Ref. 2 that $V_p = V_{mp} = V_*/3^{1/4}$ for $(DV)_{\min}$ is not a realistic choice, even though Eqs. (1) and (2) predict V_{mp} produces the maximum rate of climb, because $C_{L,mp}$ becomes too large for the airplane drag polar's validity, as shown in Fig. 1 of Ref. 2. Also, Fig. 2 of Ref. 2 shows that $V_p = (V_m)_{\max}$ would be a suitable fixed-pitch propeller design criteria since it produces $V_{\max}$ and a good rate of climb, as shown in Fig. 1 of Ref. 4. As shown in Ref. 2, any decrease in β and V_p would increase the rate of climb, but decrease the maximum speed. Another consideration is that the decrease in β and V_p increases the takeoff thrust, since $E = (3\phi P_p)/(2V_p)$ also increases (see Fig. 3 in Ref. 2).

However, if neither the rate of climb, nor the takeoff distance, are the prime consideration, then $V_p = V_{\max}$ produces the best compromise for a fixed-pitch propeller. As indicated in Fig. 2 of Ref. 2, $V_p > V_{\max}$ produces an inferior propeller with lower climb rates. The optimum $V_p = V_{\max}$ is given for $\sigma = 1$, by

$$DV = GV_p^3 + H/V_p = P_p = \eta \text{ (bhp)}$$

The resulting quartic equation is easily evaluated by the following first iteration [see Eq. (11), Ref. 3]:

$$V_{\max} = V_p \approx (P_p/G)^{1/3} [1 - (H/G)(P_p/G)^{-4/3}]^{1/3} \quad (5)$$

The resulting values for V_p and P_p allow Eq. (2) to be used to estimate the performance for $V < V_p$ and $\sigma < 1$ for the optimum fixed-pitch propeller.

References

- ¹Lowry, J. T., "Analytic V Speeds from Linearized Propeller Polar," *Journal of Aircraft*, Vol. 33, No. 1, 1996, pp. 233-235.
- ²Laitone, E. V., "Performance Prediction for Light Airplanes," *Journal of Aircraft*, Vol. 18, No. 11, 1981, pp. 988-991.
- ³Laitone, E. V., "Performance Estimation of Light Propeller Airplanes," *Journal of Aircraft*, Vol. 20, No. 6, 1983, pp. 569-571.
- ⁴Laitone, E. V., "Rate of Climb for Light Propeller Powered Airplanes," *Journal of Aircraft*, Vol. 21, No. 4, 1984, pp. 287, 288.

Reply by the Author to E. V. Laitone

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IN his Comments on Lowry's¹ Note, Laitone² cites three instances in which Lowry's equations are "identical to" equations given by Laitone³ in earlier work. We disagree, on grounds that two equations cannot be identical when they contain variables standing for essentially different physical constructs. Lowry's entire development was based, as the title of his Note indicates, on the linearized propeller polar. The propeller polar concept, linearized or not, is never mentioned in Laitone's³ article or in either of his follow-up articles^{4,5} cited in Ref. 2.

When the subject Note was submitted for publication we were not aware of Laitone's work. If Laitone's³⁻⁵ articles had

been brought to our attention they would have been cited in our Note as interesting work along similar lines, just as was done in the case of similar work by von Mises.⁶ Each of these three investigators assumed identical quadratic forms for the variation of thrust T with airspeed V :

$$T(V) = E + FV^2 \quad (1)$$

Since there is no argument over the form of the drag polar

$$D(V) = GV^2 + (H/V^2) \quad (2)$$

(though there may be disagreement over the range of speeds for which it accurately holds), the form of Eq. (1) is the precise and sole reason behind the identical forms of V -speed solutions given by the three investigators. Identity of form under varying interpretations of coefficients E and F is not, in our opinion, tantamount to the equations themselves being identical. "At night all cows are gray," said Hegel; likewise, under open-ended substitution of variables, all equations read the same: $A = B$.

The physical meaning of any one of these investigator's interpretation of coefficient E or F does differ from the others. von Mises's interpretation employs representative blade elements. Laitone's interpretation uses static thrust and maximum level flight speed at sea level, assumed to occur at the point where power available TV also peaks. Lowry's interpretation uses the linearized propeller polar parameters, slope m and intercept b . The eclectic reader is free to decide which interpretation is most reasonable, compelling, and useful to his or her purposes.

Because propellers may behave strangely at very low forward speeds and small advance ratios J , we preferred not to use static thrust as a basic parameter in our theory. It is, for instance, not quite the case, as mentioned by Laitone,² that "thrust must decrease as the velocity increases." In the regime of ordinary flight his assertion is almost certainly true (and we have used it there), but at very low (or no) forward speed, propeller blade stall characteristics may result in positive values for dC_T/dJ , and hence, possibly to increases of thrust with increased airspeed. For examples, see thrust coefficient graphs in the classic article of Hartman and Biermann.⁷

Another point of difference between the treatments of Lowry¹ and Laitone² is exemplified in the latter's assertion "the estimate of η (the propulsive efficiency) which is usually 0.85 from [maximum level flight speed to speed for maximum lift/drag ratio]." Unless supplemented with rpm data, Lowry's scheme makes no use of the concept of propeller efficiency. As written elsewhere, Lowry's^{8,9} method is a reduced description involving only one relation, the propeller polar, instead of the pair of relations afforded by the full story, graphs of thrust coefficient and power coefficient as functions of advance ratio.

A further point of difference resides in Laitone's³ statement "For a fixed-pitch propeller, Oswald has indicated that for most aircraft the best performance is obtained when the maximum or peak-power is available at the maximum velocity for steady level flight at sea level . . ." Laitone's use of this idea shows not only a difference in technique (we have found use of relations that one hopes should hold, or of speed ratios, to be much less fertile than a constructive approximate theory of propeller action), but also of basic motive. The purpose of the linear propeller polar approach to flight performance prediction is to not presume the characteristics of some ideal aircraft, or of most aircraft, but simply to give airplane owners, fleet operators, and manufacturers a useful tool for evaluating performance of the actual aircraft they have at hand. The propeller polar bootstrap approach is currently being used for that practical purpose by some small aircraft manufacturers.

References

- ¹Lowry, J. T., "Analytic V Speeds from Linearized Propeller Po-

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lar," *Journal of Aircraft*, Vol. 33, No. 1, 1996, pp. 233–235.

²Laitone, E. V., "Comment on Analytic V Speeds from Linearized Propeller Polar," *Journal of Aircraft*, Vol. 33, No. 5, 1996, pp. 1030, 1031.

³Laitone, E. V., "Performance Prediction for Light Airplanes," *Journal of Aircraft*, Vol. 18, No. 11, 1981, pp. 988–991.

⁴Laitone, E. V., "Performance Estimation for Light Propeller Airplanes," *Journal of Aircraft*, Vol. 20, No. 6, 1983, pp. 569–571.

⁵Laitone, E. V., "Rate of Climb for Light Propeller Powered Airplanes," *Journal of Aircraft*, Vol. 21, No. 4, 1984, pp. 287, 288.

⁶Von Mises, R., *Theory of Flight*, Dover, New York, 1959, Chap. 15.

⁷Hartman, E. P., and Biermann, D., "The Aerodynamic Characteristics of Full-Scale Propellers Having 2, 3, and 4 Blades of Clark Y and R.A.F. 6 Airfoil Sections," NACA TR 640, 1938, Figs. 8, 9, 20, 22, 27, and 29.

⁸Lowry, J. T., "The Bootstrap Approach to Predicting Airplane Flight Performance," *Journal of Aviation/Aerospace Education and Research*, Vol. 6, No. 1, 1995, pp. 25–33.

⁹Lowry, J. T., *Computing Airplane Performance with the Bootstrap Approach: A Field Guide*, M Press, Billings, MT, 1995, Chap. 6.